# Exercise 17

In Exercises 17 and 18, **(a)** write formulas for $f \circ g$ and $g \circ f$ and **(b)** find the domain of each.

$$f(x) = \sqrt{x+1}, \ g(x) = \frac{1}{x}$$

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**Solution**

**Part (a)**

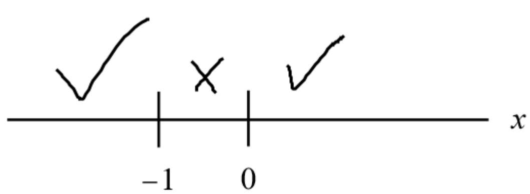
Calculate $f \circ g$ first.

$$\begin{aligned} f \circ g &= f(g(x)) \\ &= f\left(\frac{1}{x}\right) \\ &= \sqrt{\frac{1}{x}+1} \\ &= \sqrt{\frac{1+x}{x}} \end{aligned}$$

The denominator cannot be zero, and only the square root of a nonnegative number can be taken.

$$x \neq 0 \quad \text{and} \quad \frac{1+x}{x} \geq 0$$

The critical points for the inequality on the right are $x = -1$ and $x = 0$. Partition the real line at these values of $x$ and test whether the inequality is true in the intervals.




$$x \neq 0 \quad \text{and} \quad (x \leq -1 \quad \text{or} \quad x \geq 0)$$

$$x \leq -1 \quad \text{or} \quad x > 0$$

Consequently, the domain for $f \circ g$ is $(-\infty, -1] \cup (0, \infty)$.

**Part (b)**

Calculate $g \circ f$ second.

$$\begin{aligned} g \circ f &= g(f(x)) \\ &= g\left(\sqrt{x+1}\right) \\ &= \frac{1}{\sqrt{x+1}} \end{aligned}$$

The denominator cannot be zero, and only the square root of a nonnegative number can be taken.

$$x + 1 \neq 0 \quad \text{and} \quad x + 1 \geq 0$$

$$x + 1 > 0$$

$$x > -1$$

Consequently, the domain for $g \circ f$ is $(-1, \infty)$.